Effect of Dust Particles on Thermal Convection in a Ferromagnetic Fluid

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This paper deals with the theoretical investigation of the effect of dust particles on the thermal convection in a ferromagnetic fluid subjected to a transverse uniform magnetic field. For a flat ferromagnetic fluid layer contained between two free boundaries, the exact solution is obtained, using a linear stability analysis. For the case of stationary convection, dust particles and non-buoyancy magnetization have always a destabilizing effect. The critical wavenumber and critical magnetic thermal Rayleigh number for the onset of instability are also determined numerically for sufficiently large values of the buoyancy magnetization parameter M_1. The results are depicted graphically. It is observed that the critical magnetic thermal Rayleigh number is reduced because the heat capacity of the clean fluid is supplemented by that of the dust particles. The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below in the absence of dust particles. The oscillatory modes are introduced by the dust particles. A sufficient condition for the non-existence of overstability is also obtained.

Key words: Ferromagnetic Fluid; Thermal Convection; Vertical Magnetic Field; Dust Particles.

1. Introduction

Ferrohydrodynamics (FHD) deals with fluid motions influenced by strong forces of magnetic polarization. It concerns usually non-conducting liquids with magnetic properties and constitutes an entire field of physics close to magnetohydrodynamics, but still different. The major perspectives are connected with massive shocks and oscillation damping (earthquake, airbags), but the contemporary application concerned mostly seals and cooling of loudspeakers. Strong efforts have been made to synthesize stable suspensions of magnetic particles with different performances.

Experimental and theoretical physicists and engineers gave significant contributions to ferrohydrodynamics and its applications [1 – 7]. An authoritative introduction to the research on magnetic liquids has been discussed in the monograph by Rosensweig [8], which reviews several applications of heat transfer through ferrofluids, such as enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, depends on the magnetic field, temperature and density of the fluid. Any variation of these quantities can induce a change of the force distribution in the fluid. This mechanism is known as ferroconvection, which is similar to Bénard convection [9]. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of an uniform vertical magnetic field has been considered by Finlayson [10]. He explained the concept of thermo-mechanical interaction on ferrofluids. Thermoconvective stability of ferrofluids without considering buoyancy effects has been investigated by Lalas and Carmi [11], whereas Shliomis [12] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability. Schwab et al. [13] investigated experimentally Finlayson's problem in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number versus the Rayleigh number. Then, the critical Rayleigh number corresponds to a discontinuity in the slope. Later, Stiles and Kagan [14] examined the experimental problem reported in [13] and generalized Finlayson's model assuming that under a strong magnetic field, the rotational viscosity augments the shear viscosity. The thermal convection in a ferrofluid has been considered by Zebib [15], whereas the stability of a static ferrofluid under the action of an external pressure drop has been studied by Polevikov [16]. The thermal convection in a cylindrical layer of magnetic fluid has been studied

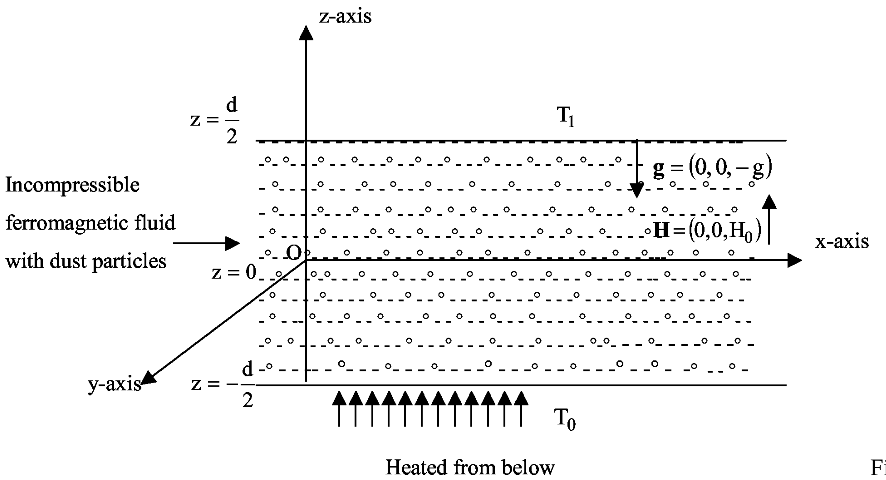

Fig. 1. Geometrical configuration.

by Lange [17]. A detailed account of magnetoviscous effects in ferrofluids has been given in a monograph by Odenbach [7].

The Bénard convection in ferromagnetic fluids has been considered by many authors [18–28]. The ferromagnetic fluid has been considered to be clean in all the above studies. In many situations the fluid is not pure but contains suspended dust particles. Saffman [29] has considered the stability of laminar flow of a dusty gas. Scanlon and Segel [30] have considered the effects of suspended particles on the onset of Bénard convection, whereas Sharma et al. [31] have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics and found that the critical Rayleigh number is reduced because of the heat capacity of the particles. The separate effects of suspended particles, rotation and solute gradient on the thermal instability of fluids through a porous medium have been discussed by Sharma and Sharma [32]. The suspended particles were thus found to destabilize the layer. Palaniswamy and Purushotham [33] have studied the stability of shear flow of stratified fluids with fine dust and found the fine dust increases the region of instability. On the other hand, the multiphase fluid systems are concerned with the motion of a liquid or gas containing inmiscible inert identical particles. Of all multiphase fluid systems observed in nature, blood flows in arteries, flow in rocket tubes, dust in gas cooling systems to enhance the heat transfer, movement of inert solid particles in the atmosphere, sand or other particles in sea or ocean beaches are the most common examples of multiphase fluid systems. Naturally studies of these systems are mathematically interesting and physically useful for various good reasons. The effect of dust particles on non-magnetic fluids has been investigated by many authors [34–37]. The main result of all these studies is that dust particles are destabilizing and the specific heat of fluid being greater than the specific heat of particles is a sufficient condition for the non-existence of overstability.

In view of the above investigations, and keeping in mind the importance of ferromagnetic fluids, it is attempted to discuss the effect of dust particles on thermal convection in a ferromagnetic fluid subjected to a vertical magnetic field. This problem has, to the best of our knowledge, not been investigated yet. The present study can serve as a theoretical support for experimental investigations, e.g. evaluating the influence of impurifications in a ferromagnetic fluid on thermal convection phenomena.

2. Mathematical Formulation of the Problem

We consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible ferromagnetic fluid with dust particles heated and soluted from below. A uniform magnetic field H_0 acts along the vertical z-axis. The temperature at the bottom and top surfaces $z = \mp d/2$ are T_0 and T_1, and a uniform temperature gradient $\beta = |\mathrm{d}T/\mathrm{d}z|$ is maintained (see Fig. 1). Both the boundaries are taken to be perfect conductors of heat. A gravitational field $\boldsymbol{g} = (0, 0, -g)$ pervades the system.

The mathematical equations governing the motion of a ferromagnetic fluid are as follows:

Continuity equation for an incompressible ferromagnetic fluid:

$$\nabla\cdot\boldsymbol{q} = 0. \tag{1}$$

Momentum equation:

$$\rho_0\left[\frac{\partial}{\partial t}+(\boldsymbol{q}\cdot\nabla)\right]\boldsymbol{q} = -\nabla p+\rho\boldsymbol{g}+KN(\boldsymbol{q}_{\mathrm{d}}-\boldsymbol{q}) \\ +\nabla\cdot(\boldsymbol{HB})+\mu\nabla^2\boldsymbol{q}, \tag{2}$$

where ρ, ρ_0, $\boldsymbol{q}$, μ and p are the fluid density, reference density, velocity, dynamic viscosity (constant) and pressure of ferromagnetic fluid, respectively; t, $\boldsymbol{H}$, $\boldsymbol{B}$ and $\boldsymbol{M}$ denote, respectively, the time, magnetic field, magnetic induction and magnetization. $\boldsymbol{q}_{\mathrm{d}}(\boldsymbol{x},t)$ and $N(\boldsymbol{x},t)$ denote the velocity and number density of the dust particles, respectively. $\boldsymbol{x} = (x,y,z)$ and $K = 6\pi\mu\eta$, η being the particle radius, is the Stokes drag coefficient. Assuming a uniform particle size, a spherical shape, and small relative velocities between the fluid and dust particles, the presence of dust particles adds an extra force term in the equations of motion (2), proportional to the velocity difference between the dust particles and the fluid. Two additional simplifications are assumed in (2): we assume that the viscosity is isotropic and independent of the magnetic field. Both approximations simplify the analysis without changing the ultimate conclusion. We also use the Boussinesq approximation, which means that density changes are disregarded in all terms except the gravitational body force term.

Density equation of state:

$$\rho = \rho_0[1-\alpha(T-T_{\mathrm{a}})], \tag{3}$$

where α is the thermal expansion coefficient, T_{a} is the average temperature given by $T_{\mathrm{a}} = (T_0+T_1)/2$, where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the ferromagnetic fluid layer.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles is neglected. Inter-particle reactions are also ignored since we assume that the distances between particles are quite large compared with their diameters. The effects due to pressure and gravity on the particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity of the dust particles, under the above assumptions, are

$$mN\left[\frac{\partial\boldsymbol{q}_{\mathrm{d}}}{\partial t}+(\boldsymbol{q}\cdot_{\mathrm{d}}\nabla)\boldsymbol{q}_{\mathrm{d}}\right] = KN(\boldsymbol{q}-\boldsymbol{q}_{\mathrm{d}}), \tag{4}$$

$$\frac{\partial N}{\partial t}+\nabla\cdot(N\boldsymbol{q}_{\mathrm{d}}) = 0. \tag{5}$$

The temperature equation for an incompressible ferromagnetic fluid in the presence of dust particles is

$$\left[\rho_0 C_{V,H}-\mu_0\boldsymbol{H}\cdot\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,\boldsymbol{H}}\right]\frac{\mathrm{D}T}{\mathrm{D}t} \\ +\mu_0 T\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,\boldsymbol{H}}\frac{\mathrm{D}\boldsymbol{H}}{\mathrm{D}t} \\ +mNC_{\mathrm{pt}}\left(\frac{\partial}{\partial t}+\boldsymbol{q}_{\mathrm{d}}\cdot\nabla\right)T = K_1\nabla^2 T, \tag{6}$$

where $C_{V,H}$, C_{pt}, K_1, T and μ_0 are the specific heat at constant volume and magnetic field, specific heat of dust particles, thermal conductivity, temperature and magnetic permeability, respectively.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla\cdot\boldsymbol{B} = 0, \quad \nabla\times\boldsymbol{H} = \boldsymbol{0}. \tag{7}$$

In the Chu formulation of electrodynamics [38], the magnetic field $\boldsymbol{H}$, magnetization $\boldsymbol{M}$ and the magnetic induction $\boldsymbol{B}$ are related by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H}+\boldsymbol{M}). \tag{8}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H}M(H,T). \tag{9}$$

The magnetic equation of state is linearized about the magnetic field, H_0, and an average temperature, T_{a}, to become

$$M = M_0+\chi(H-H_0)-K_2(T-T_{\mathrm{a}}), \tag{10}$$

where H_0 is the uniform magnetic field of the fluid layer when placed in an external magnetic field $\boldsymbol{H} = \hat{\boldsymbol{k}}H_0^{\mathrm{ext}}$, $\hat{\boldsymbol{k}}$ is the unit vector in the z-direction, $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_0,T_{\mathrm{a}}}$ the magnetic susceptibility, $K_2 =$

$-\left(\frac{\partial M}{\partial T}\right)_{H_0,T_a}$ the pyromagnetic coefficient, H magnitude of $\boldsymbol{H}$ and $M_0 = M(H_0,T_a)$.

The basic state is assumed to be quiescent and is given by

$$
\begin{aligned}
&\boldsymbol{q}=\boldsymbol{q}_b=\boldsymbol{0}, \quad \boldsymbol{q}_d=(\boldsymbol{q}_d)_b=\boldsymbol{0}, \quad \rho=\rho_b(z),\\
&p=p_b(z), \quad T=T_b(z)=-\beta z+T_a,\\
&\beta=\frac{T_1-T_0}{d}, \quad \boldsymbol{H}_b=\left[H_0+\frac{K_2(T_b-T_a)}{1+\chi}\right]\boldsymbol{k},\\
&\boldsymbol{M}_b=\left[M_0-\frac{K_2(T_b-T_a)}{1+\chi}\right]\boldsymbol{k}, \quad N=N_0,\\
&H_0+M_0=H_0^{\text{ext}}.
\end{aligned} \tag{11}
$$

Only the spatially varying parts of $\boldsymbol{H}_0$ and $\boldsymbol{M}_0$ contribute to the analysis, so that the direction of the external magnetic field is unimportant and the convection is the same whether the external magnetic field is parallel or antiparallel to the gravitational force.

3. The Perturbation Equations and Normal Mode Analysis Method

We shall analyze the stability of the basic state by introducing the following perturbations:

$$
\begin{aligned}
&\boldsymbol{q}=\boldsymbol{q}_b+\boldsymbol{q}', \quad \boldsymbol{q}_d=(\boldsymbol{q}_d)_b+\boldsymbol{q}_1', \quad p=p_b(z)+p',\\
&\rho=\rho_b+\rho', \quad T=T_b(z)+\theta, \quad \boldsymbol{H}=\boldsymbol{H}_b(z)+\boldsymbol{H}',\\
&\boldsymbol{M}=\boldsymbol{M}_b(z)+\boldsymbol{M}',
\end{aligned} \tag{12}
$$

where $\boldsymbol{q}'=(u,v,w)$, $\boldsymbol{q}_1'=(\ell,r,s)$, p', ρ', θ, $\boldsymbol{H}'$ and $\boldsymbol{M}'$ are perturbations in the ferromagnetic fluid velocity, particle velocity, pressure, density, temperature, magnetic field and magnetization. These perturbations are assumed to be small. Then the linearized perturbation equations become

$$L_1\rho_0\frac{\partial u}{\partial t}=L_1\left[-\frac{\partial p'}{\partial x}+\mu_0(M_0+H_0)\frac{\partial H_1'}{\partial z}+\mu\nabla^2 u\right]-mN_0\frac{\partial u}{\partial t}, \tag{13}$$

$$L_1\rho_0\frac{\partial v}{\partial t}=L_1\left[-\frac{\partial p'}{\partial y}+\mu_0(M_0+H_0)\frac{\partial H_2'}{\partial z}+\mu\nabla^2 v\right]-mN_0\frac{\partial v}{\partial t}, \tag{14}$$

$$L_1\rho_0\frac{\partial w}{\partial t}=L_1\left[-\frac{\partial p'}{\partial z}+\mu_0(M_0+H_0)\frac{\partial H_3'}{\partial z}+\mu\nabla^2 w-\mu_0K_2\beta H_3'+\frac{\mu_0K_2^2\beta}{(1+\chi)}\theta+g\alpha\rho_0\theta\right]-mN_0\frac{\partial w}{\partial t}, \tag{15}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{16}$$

$$
\begin{aligned}
&L_1\left[\left\{\rho C_1+mN_0C_{pt}\right\}\frac{\partial\theta}{\partial t}-\mu_0T_0K_2\frac{\partial}{\partial t}\left(\frac{\partial\Phi'}{\partial z}\right)\right]\\
&=L_1\left[K_1\nabla^2\theta+\left\{\rho C_1\beta-\frac{\mu_0T_0K_2^2\beta}{(1+\chi)}\right\}w\right]\\
&\quad+mN_0\beta C_{pt}w,
\end{aligned} \tag{17}
$$

where

$$\rho C_1=\rho_0C_{V,H}+\mu_0K_2H_0,\ L_1=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right). \tag{18}$$

Equations (9) and (10) yield

$$
\left.\begin{aligned}
&H_3'+M_3'=(1+\chi)H_3'-K_2\theta\\
&H_i'+M_i'=\left(1+\frac{M_0}{H_0}\right)H_i',\ (i=1,2)
\end{aligned}\right\}, \tag{19}
$$

where we have assumed $K_2\beta d\ll(1+\chi)H_0$, as the analysis is restricted to a physical situation in which the magnetization induced by temperature variations is small compared to that induced by the external magnetic field. Equation (7b) means we can write $\boldsymbol{H}'=\nabla\Phi'$, where Φ' is the perturbed magnetic potential.

Eliminating u, v, p' in (13), (14), and (15), using (16), we obtain

$$
\begin{aligned}
&\left\{\left(\rho_0\frac{\partial}{\partial t}-\mu\nabla^2\right)L_1+mN_0\frac{\partial}{\partial t}\right\}\nabla^2 w=\\
&L_1\left\{-\mu_0K_2\beta\left(\nabla_1^2\frac{\partial\Phi'}{\partial z}\right)+\rho_0g\alpha(\nabla_1^2\theta)\right.\\
&\left.+\frac{\mu_0K_2^2\beta}{(1+\chi)}(\nabla_1^2\theta)\right\}.
\end{aligned} \tag{20}
$$

From (19), we have

$$(1+\chi)\frac{\partial^2\Phi'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi'-K_2\frac{\partial\theta}{\partial z}=0. \tag{21}$$

The normal mode solution of all dynamical variables can be written as

$$f'(x,y,z,t) = f(z,t)\exp \mathrm{i}(k_x x + k_y y), \tag{22}$$

where k_x, k_y are the wavenumbers along the x- and y-directions, respectively, and $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wavenumber. For functions with this dependence on x, y, and t, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -k^2$ and $\nabla^2 = \frac{\partial^2}{\partial z^2} - k^2$.

Equations (20), (17), and (21), using (22), become

$$\left[\left\{\rho_0 \frac{\partial}{\partial t} - \mu\left(\frac{\partial^2}{\partial z^2} - k^2\right)\right\} L_1 + mN_0 \frac{\partial}{\partial t}\right] \cdot \left(\frac{\partial^2}{\partial z^2} - k^2\right) W = L_1\left[\frac{\mu_0 K_2 \beta}{(1+\chi)}\left\{(1+\chi)\frac{\partial \Phi}{\partial z} - K_2 \Theta\right\} k^2 - \rho_0 g \alpha k^2 \Theta\right], \tag{23}$$

$$L_1\left[(\rho C_1 + mN_0 C_{\mathrm{pt}})\frac{\partial \Theta}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t}\left(\frac{\partial \Phi}{\partial z}\right)\right] = L_1\left[K_1\left(\frac{\partial^2}{\partial z^2} - k^2\right)\Theta + \left(\rho C_1 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{1+\chi}\right) W\right] + mN_0 C_{\mathrm{pt}} \beta W, \tag{24}$$

$$(1+\chi)\frac{\partial^2 \Phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right) k^2 \Phi - K_2 \frac{\partial \Theta}{\partial z} = 0. \tag{25}$$

Equations (23) – (25) give the following dimensionless equations

$$\left[L_1^*\left(\frac{\partial}{\partial t^*} - (D^2 - a^2)\right) + f\frac{\partial}{\partial t^*}\right](D^2 - a^2) W^* = aR^{1/2} L_1^* [M_1 D\Phi^* - (1+M_1) T^*], \tag{26}$$

$$L_1^* P_{\mathrm{r}}\left[(1+h)\frac{\partial T^*}{\partial t^*} - M_2 \frac{\partial}{\partial t^*}(D\Phi^*)\right] = L_1^*(D^2 - a^2) T^* + aR^{1/2}[L_1^*(1 - M_2) + h] W^*, \tag{27}$$

$$D^2 \Phi^* - a^2 M_3 \Phi^* - DT^* = 0, \tag{28}$$

where the following non-dimensional parameters are introduced:

$$t^* = \frac{\nu t}{d^2}, \quad W^* = \frac{d}{\nu} W, \quad \Phi^* = \frac{(1+\chi) K_1 a R^{1/2}}{K_2 \rho C_1 \beta \nu d^2} \Phi,$$

$$R = \frac{g\alpha\beta d^4 \rho C_1}{\nu K_1}, \quad T^* = \frac{K_1 a R^{1/2}}{\rho C_1 \beta \nu d} \Theta,$$

$$a = kd, \quad z^* = \frac{z}{d}, \quad D = \frac{\partial}{\partial z^*}, \quad P_r = \frac{\nu}{K_1} \rho C_1,$$

$$M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha \rho_0 g}, \quad M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_1},$$

$$M_3 = \frac{\left(1 + \frac{M_0}{H_0}\right)}{(1+\chi)}, \ \tau = \frac{m\nu}{Kd^2}, \ L_1^* = \left(\tau \frac{\partial}{\partial t^*} + 1\right),$$

$$f = \frac{mN_0}{\rho_0}, \quad h = \frac{mN_0 C_{\mathrm{pt}}}{\rho C_1}. \tag{29}$$

Here M_3 and h denote, respectively, non-buoyancy magnetization and dust particle parameters. The parameter M_3 measures the departure of linearity in the magnetic equation of state. Values from one ($M_0 = \chi H_0$) to higher values are possible for the usual equations of state.

4. Exact Solution for Free Boundaries

We consider the case where both boundaries are free, as well as perfect conductors of heat. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our present analysis. Here we consider the case of an infinite magnetic susceptibility χ and we neglect the deformability of the horizontal surfaces. Thus the exact solution of the system (26) – (28), subject to the boundary conditions

$$W^* = D^2 W^* = T^* = D\Phi^* = 0 \text{ at } z = \pm\frac{1}{2}, \tag{30}$$

is written in the form

$$W^* = A_1 \mathrm{e}^{\sigma t^*} \cos \pi z^*, \quad T^* = B_1 \mathrm{e}^{\sigma t^*} \cos \pi z^*,$$
$$D\Phi^* = C_1 \mathrm{e}^{\sigma t^*} \cos \pi z^*, \ \Phi^* = \left(\frac{C_1}{\pi}\right) \mathrm{e}^{\sigma t^*} \sin \pi z^*, \tag{31}$$

where A_1, B_1, C_1 are constants and σ is the growth rate which is, in general, a complex constant.

Substituting (31) in (26) – (28) and dropping asterisks for convenience, we get the equations

$$\left\lfloor\left\{\left(\sigma + (\pi^2 + a^2)\right)(1+\tau\sigma) + f\sigma\right\}(\pi^2 + a^2)\right\rfloor A_1 - \left\lfloor aR^{1/2}(1+\tau\sigma)(1+M_1)\right\rfloor B_1 + \left\lfloor aR^{1/2} M_1 (1+\tau\sigma)\right\rfloor C_1 = 0, \tag{32}$$

$$\left\lfloor aR^{1/2}\{h+(1-M_2)(1+\tau\sigma)\}\right\rfloor A_1$$
$$-\left\lfloor\{(\pi^2+a^2)+P_r(1+h)\sigma\}((1+\tau\sigma)\right\rfloor B_1$$
$$+[P_r M_2\sigma(1+\tau\sigma)]C_1=0, \tag{33}$$
$$-\pi^2 B_1+(\pi^2+a^2M_3)C_1=0. \tag{34}$$

For the existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1 in (32)–(34) must vanish. This determinant on simplification yields

$$-\mathrm{i}T_3\sigma_1^3-T_2\sigma_1^2+\mathrm{i}T_1\sigma_1+T_0=0. \tag{35}$$

Here

$$T_3=\tau_1 bL_2, \tag{36}$$
$$T_2=b\{\tau_1 b(L_0+L_2)+(1+f)L_2\}, \tag{37}$$
$$T_1=\left\{b^2\{L_2+L_0(b\tau_1+1+f)\}\right.\\ \left.-\tau_1x_1R_1(1-M_2)L_3\right\}, \tag{38}$$
$$T_0=\left\lfloor b^3 l_0-x_1R_1L_3\{h+(1-M_2)\}\right\rfloor, \tag{39}$$

where

$$R_1=\frac{R}{\pi^4},\quad x_1=\frac{a^2}{\pi^2},\quad \mathrm{i}\sigma_1=\frac{\sigma}{\pi^2},\quad \tau_1=\tau\pi^2,$$
$$b=(1+x_1),\quad L_0=(1+x_1M_3),$$
$$L_2=P_r[\{(1-M_2)+x_1M_3\}+L_0h],\ \text{and}$$
$$L_3=(1+x_1M_3+x_1M_3M_1). \tag{40}$$

5. The Case of Stationary Convection

When the instability begins as stationary convection (and $M_2\cong 0$), the marginal state will be characterized by $\sigma_1=0$. Then the Rayleigh number is given by

$$R_1=\frac{(1+x_1)^3(1+x_1M_3)}{x_1h_1\{1+x_1(1+M_1)M_3\}}, \tag{41}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wavenumber x_1, the non-buoyancy magnetization parameter M_3, the buoyancy magnetization parameter M_1 and the dust particles parameter h_1. Here we put $h_1=(1+h)$. In the absence of dust particles, the value of h_1 is one. Since the marginal state dividing stability from instability is stationary, this means that at the onset of instability there is no relative velocity between particles and fluid and hence no particle drag on the fluid. Therefore the critical Rayleigh number is reduced solely because the heat capacity of the clean fluid is supplemented by that of the suspended (dust) particles. This explains the physics and the role of dust parameter.

To investigate the effects of non-buoyancy magnetization and dust particles, we examine the behaviour of $\frac{\mathrm{d}R_1}{\mathrm{d}M_3}$ and $\frac{\mathrm{d}R_1}{\mathrm{d}h_1}$ analytically. Equation (41) yields

$$\frac{\mathrm{d}R_1}{\mathrm{d}M_3}=\frac{M_1(1+x_1)^3}{h_1\{1+x_1M_3(1+M_1)\}^2}, \tag{42}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}h_1}=\frac{(1+x_1)^3(1+x_1M_3)}{x_1h_1^2\{1+x_1(1+M_1)M_3\}}. \tag{43}$$

This shows that, for a stationary convection, the non-buoyancy magnetization and dust particles are found to have destabilizing effects on the system.

For M_1 sufficiently large, we obtain the results for the magnetic mechanism

$$N=R_1M_1=\frac{(1+x_1)^3(1+x_1M_3)}{x_1^2h_1M_3}, \tag{44}$$

where N is the magnetic thermal Rayleigh number.

For very large values of M_3, (44) reduces to

$$N=\frac{(1+x_1)^3}{x_1h_1}=R_1 \quad \text{(in the absence of magnetic parameters).}$$

Thus for stationary convection, the ferromagnetic fluid behaves like an ordinary fluid for very large values of the magnetic parameters M_3 and M_1.

As a function of x_1, N, given by (44), attains its minimum when

$$2M_3x_1^2-(M_3-1)x_1-2=0. \tag{45}$$

The values of critical wavenumber for the onset of instability are determined numerically, using the Newton-Raphson method, by the condition $\frac{\mathrm{d}N}{\mathrm{d}x_1}=0$. With x_1 determined as a solution of (45), (44) will give the required critical magnetic thermal Rayleigh number N_c. The critical magnetic thermal Rayleigh number depends on the magnetization parameter M_3 and dust particles parameter h_1. Values of N_c determined in this way for various values of M_3 and h_1 are given in Table 1, and the results are illustrated in Figs. 2 and 3.

Figures 2 and 3 represent plots of the critical magnetic thermal Rayleigh number (N_c) versus M_3 (for various values of h_1), and h_1 (for various values of M_3),

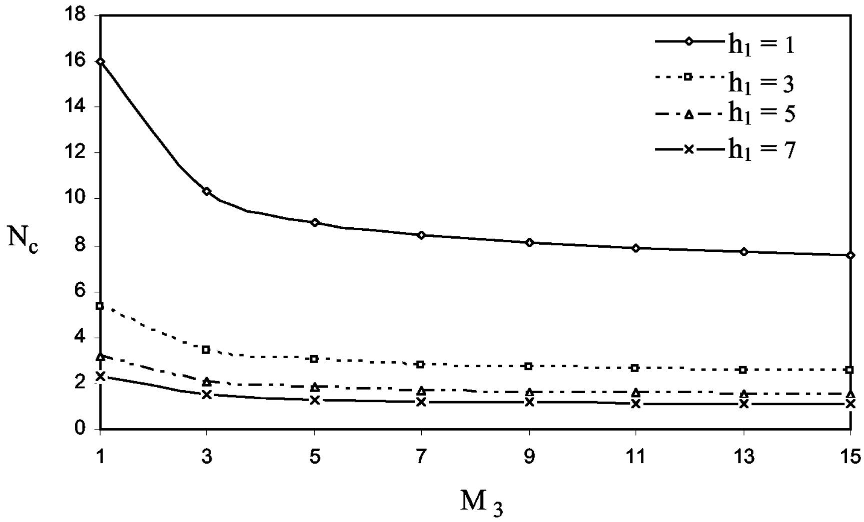

Fig. 2. The variation of the critical magnetic Rayleigh number (N_c) with the non-buoyancy magnetization parameter (M_3).

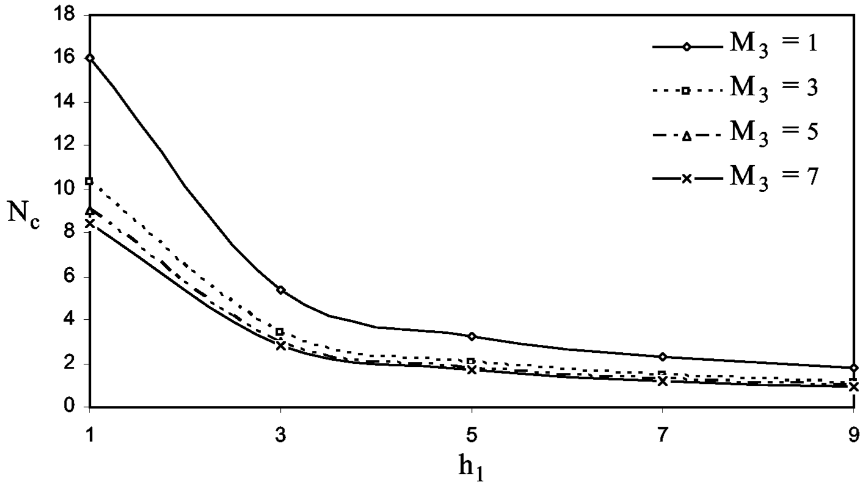

Fig. 3. The variation of the critical magnetic Rayleigh number (N_c) with the dust particles parameter (h_1).

Table 1. Critical thermal Rayleigh numbers and wavenumbers of the unstable modes at marginal stability for the onset of stationary convection.

M_3	x_c	$h_1 = 1$ N_c	$h_1 = 3$ N_c	$h_1 = 5$ N_c	$h_1 = 7$ N_c	$h_1 = 9$ N_c
1	1.00	16.00	5.33	3.20	2.29	1.78
3	0.77	10.32	3.44	2.06	1.47	1.15
5	0.69	9.02	3.01	1.80	1.29	1.00
7	0.65	8.43	2.81	1.69	1.20	0.94
9	0.62	8.09	2.70	1.62	1.16	0.90
11	0.60	7.86	2.62	1.57	1.12	0.87
13	0.59	7.70	2.57	1.54	1.10	0.86
15	0.58	7.58	2.53	1.52	1.08	0.84

respectively. Figures 2 and 3 illustrate that as magnetization parameter M_3 and dust particle parameter h_1 increase, the critical magnetic Rayleigh number N_c decreases. Therefore lower values of N_c are needed for onset of convection with an increase in M_3 and h_1, hence justifying the destabilizing effects of magnetization and dust particles. It is also observed from Table 1 and Figs. 2 and 3 that in the absence of dust particles ($h_1 = 1$), the critical magnetic thermal Rayleigh number is very high, however in the presence of dust particles ($h_1 > 1$), the critical magnetic thermal Rayleigh number is reduced, because of the specific heat of the dust particles. Also, it is clear from Table 1 and Fig. 2 that, as M_3 becomes large, it has less influence on the value of N_c.

6. The Case of Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on the stability problem due to the presence of dust particles and the magnetization parameter. Equating the imaginary parts of (35), we obtain

$$\sigma_1\{b^2L_2+b^2L_0(b\tau_1+1+f) \\ -\tau_1x_1R_1(1-M_2)L_3-\sigma_1^2b\tau_1L_2\}=0. \quad (46)$$

It is evident from (46) that σ_1 may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of dust particles we obtain the above result as

$$\sigma_1[\{P_\mathrm{r}[(1-M_2)+x_1M_3]+(1+x_1M_3)\}]=0. \quad (47)$$

Here the quantity inside the brackets is positive definite because the typical values of M_2 are $+10^{-6}$ [10]. Hence

$$\sigma_1=0, \quad (48)$$

which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of dust particles. Thus from (46), we conclude that the oscillatory modes are introduced due to the presence of the dust particles.

7. The Case of Overstability

The present section is devoted to find the possibility that instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (35) will admit solutions with real σ_1.

Equating real and imaginary parts of (35) and eliminating R_1 between them, we obtain

$$A_1\sigma_1^2+A_0=0, \quad (49)$$

where

$$A_1=\tau_1[\tau_1bL_0(1-M_2)+b\tau_1(1-M_2)L_2 \\ +L_2\{f(1-M_2)-h\}], \quad (50)$$

$$A_0=\lfloor\tau_1b^2L_0h+bL_2\{h+(1-M_2)\} \\ +bL_0\{h+(1-M_2)\} \\ +bL_0f\{h+(1-M_2)\}\rfloor. \quad (51)$$

Since σ_1 is real for overstability, both values of σ_1 are positive. But σ_1^2 is always negative if A_1 is positive (because $A_0>0$). It is clear from (50) that A_1 is positive if

$$f(1-M_2)>h, \quad (52)$$

which implies that

$$(\rho_0C_{V,H}+\mu_0K_2H_0)(1-M_2)>\rho_0C_\mathrm{pt}. \quad (53)$$

Thus, for $(\rho_0C_{V,H}+\mu_0K_2H_0)(1-M_2)>\rho_0C_\mathrm{pt}$, overstability cannot occur and the principle of the exchange of stabilities is valid. Hence the above condition is a sufficient condition for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. Thus the pyromagnetic coefficient K_2 has a significant role in developing a sufficient condition for the non-existence of overstability. In the absence of a magnetic field (and hence in the absence of magnetic parameters. i.e. a non-magnetic fluid) the above condition, as expected, reduces to $C_V>C_\mathrm{pt}$, i.e. the specific heat of a fluid at constant volume is greater than the specific heat of dust particles, which agrees well with the results obtained earlier [34–37].

8. Discussion of Results and Conclusions

The effect of dust particles on a ferromagnetic fluid heated from below in the presence of an uniform vertical magnetic field has been studied. We have investigated the effects of magnetization and dust particles on the onset of instability. The principal conclusions from the analysis of this paper are as under:

(i) For the case of stationary convection, the magnetization and dust particles hasten the onset of convection as is evident from (42) and (43).

(ii) Thus for stationary convection, the ferromagnetic fluid behaves like an ordinary fluid with very large values of the magnetic parameters M_3 and M_1.

(iii) The critical wavenumbers and critical magnetic thermal Rayleigh numbers for the onset of instability are also determined numerically for sufficiently large values of the buoyancy magnetic parameter M_1, and the results are depicted graphically. The effects of governing parameters on the stability of the system are discussed below.

- Table 1, Figs. 2 and 3 also lead to the conclusion that magnetization and dust particles have always a destabilizing effect. Therefore, lower values of N_c are needed for onset of convection with an increase in M_3 and h_1.
- We see that the critical stability parameter, N_c is reduced in the presence of dust particles because the heat capacity of the clean fluid is supplemented by that

of the suspended (dust) particles. Also, it is clear from Table 1 and Fig. 2 that as M_3 becomes large it has less influence on the value of N_c. The destabilizing effect of dust particles on non-magnetic fluid is accounted by many authors [30 – 37] and is found to be valid for a ferromagnetic fluid also.

(iv) The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below in the absence of dust particles. The oscillatory modes are introduced due to the presence of the dust particles, which were non-existent in their absence.

(v) The condition $(\rho_0 C_{V,H} + \mu_0 K_2 H_0)(1 - M_2) > \rho_0 C_{\text{pt}}$ is sufficient for the non-existence of overstability. In a non-magnetic fluid, the above condition, as expected, reduces to $C_V > C_{\text{pt}}$, i.e. the specific heat of the fluid at constant volume is greater than the specific heat of the dust particles.

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